Principles of Macroeconomics: Investment and Savings

Class 7

Alex Houtz

September 8, 2025

University of Notre Dame

Overview

- Announcements:
  - LC 10, GH 10 due Friday at 11:59pm
- Topics:
  - Investment
  - Savings
  - The market for savings and investment
- Readings:
  - Chapter 10.1, chapters 10.2-10.3

Capital

- The last two weeks, we used the following production function:

$$Y = AK^{\alpha}L^{1-\alpha}$$

  where $K \equiv$ capital.
- How was capital supplied? We assumed the supply was fixed at $\bar{K}$
- Now, we will relax this assumption

Where Does Capital Come From

- Recall that capital is the stock of productive assets:
  - Factories, robots, trucks, highways, etc.
- We only get capital through investment
  - The purchase of goods and services by businesses and households that add to the capital stock
- Important: this is "real" investment – we are not referring to financial transactions

Capital Accumulation

- So we gain capital by investing – but capital also wears out
- How can we model this?

$$K_{t+1} = K_t - \delta K_t + I_t$$

where $K_{t+1} \equiv$ capital tomorrow, $K_t \equiv$ capital today, $\delta \equiv$ the "depreciation rate", and $I_t \equiv$ investment

- When does the capital stock increase? When $I_t > \delta K_t$

Well, OK, Where Does Investment Come From?

- Investment comes from savings
- Define savings as follows:
  - National Saving is private saving + government saving
  - In the real world, capital markets go across borders. So if we invest more than we save, then we borrow from the rest of the world.

The Return of GDP

- Remember GDP?

$$Y = C + I + G + (x - im)$$

- Rearrange:

$$Y - C - G = I + (x - im)$$

- Here, the left-hand side represents what is left from GDP after total consumption (by either households or the government). By definition, these are national savings, S!
- So savings is equal to investment plus the trade balance:

$$S = I + (x - im)$$

A Closed Economy

- Suppose the economy is closed
  - This just means that there is no international trade
- Then $x,\ im = 0$ and $S = I$
- A country then can only invest as much as it saves

An Open Economy

- Now suppose the economy is open
  - Now we allow trade
- There are two cases:
  (1) Exports equals imports – then $x - im = 0$ and $S = I$ again
  (2) Exports don't equal imports – then $x - im \neq 0$ and $S \neq I$
- In case (2), the trade balance matters for analysis

Capital Flows

- But how does that make sense?
- $x > im$ implies that a country is shipping goods out of the country more than into it
  - Then foreign buyers are selling us their assets – so financial capital flows out
  - This is like us saving in foreign assets
- $x < im$ implies that a country is shipping goods into the country more than out of it
  - Then foreign buyers are purchasing our assets – financial capital flows in
  - This is like us borrowing from foreign lenders
- Call the flow of assets the net capital inflow (NCI). Then based on the above:

$$NCI = im - x = -(\text{trade balance})$$

- So another way to view investment:

$$I = S + NCI$$

Going Further into Savings

- We can split national savings, S, into two parts: private and government

$$S = S_{private} + S_{gov}$$

- Let's look at what's called the government budget constraint:

$$G + TR + S_{gov} = T$$

- where $G \equiv$ government spending, $TR \equiv$ transfer payments (like social security), and $T \equiv$ tax revenue
  - $S_{gov} > 0 \longrightarrow$ budget surplus
  - $S_{gov} < 0 \longrightarrow$ budget deficit

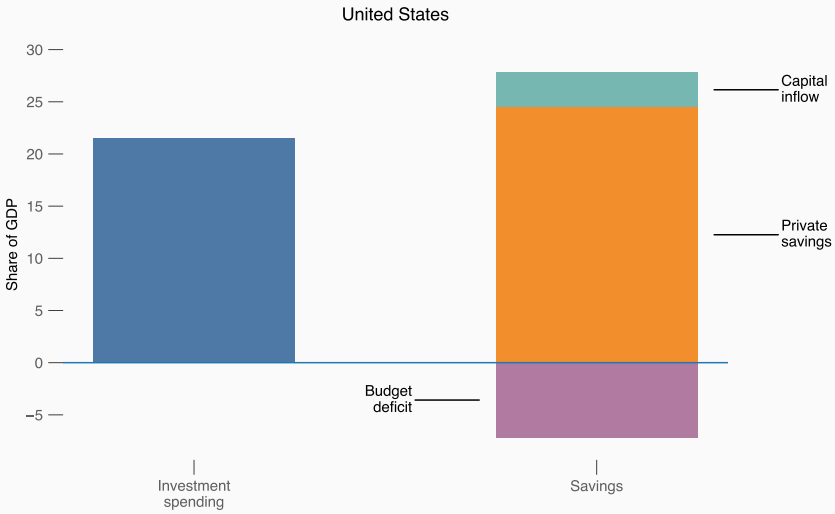
- Households have their budget constraint:

$$Y + TR = T + S_{private} + C$$

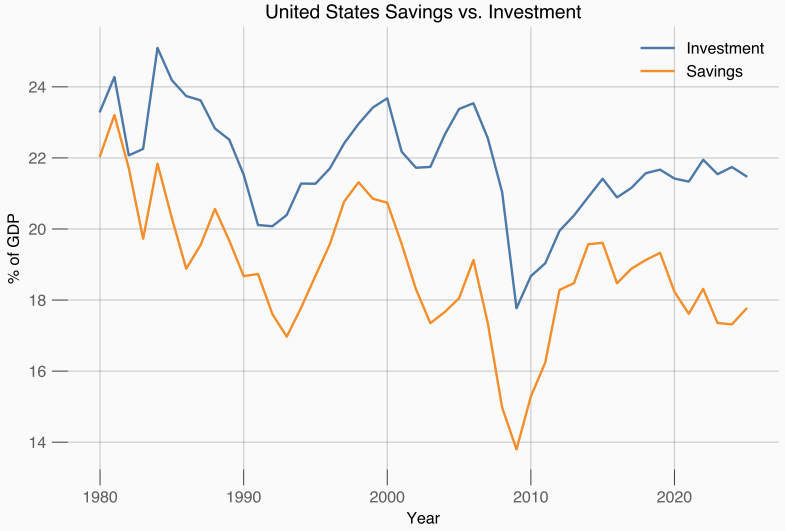
- Then from the definition of National Savings:

$$\begin{aligned} S &= Y - C - G \\ &= Y - C - G + TR - TR + T - T \\ &= \underbrace{Y + TR - T - C}_{S_{private}} + \underbrace{T - G - TR}_{S_{gov}} \end{aligned}$$

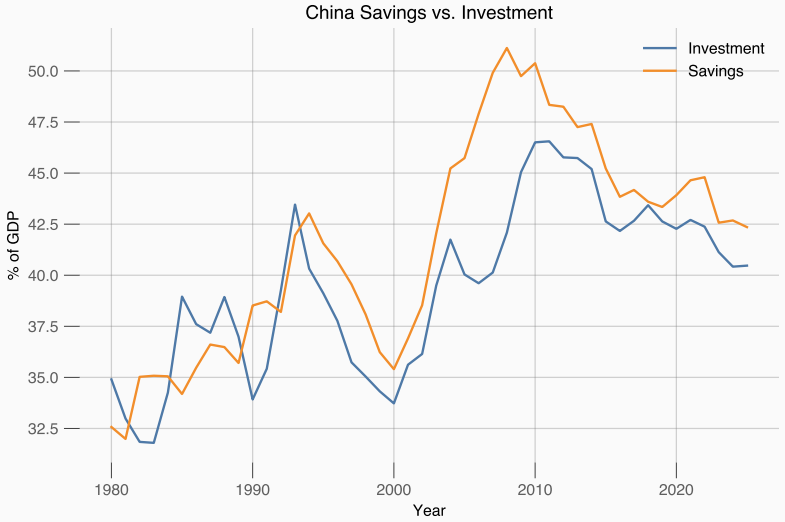
Breaking this Down for the United States



US Savings vs. Investment Over Time



China Savings vs. Investment Over Time



The Market for Loanable Funds

- Financial markets match savers and investors
- Demand for Loanable Funds:
  - "Demand for investment"
  - Depends on the interest rate
  - Interest rates are like a "price of investment"
- Supply of Loanable Funds:
  - "Supply of savings"
  - Depends on the interest rate
  - Interest rates represent the return to saving

Investment Demand

- Investment depends negatively on the interest rate: $\frac{\partial I}{\partial r} < 0$
  - Basically, as the interest rate increases, the amount of investment demanded falls

(1) Borrowing
  - If a firm borrows to create investment, loan repayment depends on the interest rate r
  - High r makes it less likely a firm will want to borrow

(2) Discounting of the future
  - We could invest money today and get a future return in a project
  - But, there's a time value of money – \$1 today is worth more than \$1 tomorrow
  - To calculate how valuable that investment is, we calculate the present discounted value (PDV)
  - PDV is lower when the outside interest rate, r, is high

(3) Opportunity Cost
  - Related to (2), funds used for investment could be used in another way – stock market, bonds, etc.
  - High r increases opportunity cost of investment

Savings Supply

$$S = S_{private} + S_{gov}$$

- Private savings decisions reflect a trade-off between consumption today and saving
  - We like consumption today
  - We want a return for giving up consumption today – a return on our savings
  - The interest rate is that return
  - We would give up \$1 of consumption today to get $\$1 \times (1 + r)$ of consumption tomorrow
- Government savings decisions we usually take as exogenous – that is, determined by factors outside the model

$$\underbrace{S_{private}(r) + NCI}_{\text{Supply}} = \underbrace{I(r) - S_{gov}}_{\text{Demand}}$$

- Supply consists of private savings, $S_{private}(r)$, and foreign capital inflows (NCI)
- Demand consists of investment $I(r)$ and government savings (S_{gov})
  - S_{gov} is subtracted because if the government demands savings, it is *borrowing*, and its savings is thus negative
- For the reasons on the previous slides supply slopes up, demand slopes down
- As usual, we set supply equal to demand to solve for equilibrium

Practice Problem – Accounting

Country A has:

$$Y = 2000, \quad C = 1350, \quad G = 300, \quad x = 250, \quad im = 200.$$

1. Compute:
   (a) National saving S
   (b) Trade balance $TB \equiv x - im$
   (c) Investment I
   (d) Net capital inflow NCI, and state whether capital flows in or out
2. Suppose next year domestic investment rises by 30, while Y, C, G are unchanged. What must happen to the trade balance and to NCI?
3. Now suppose the government raises spending by 40 and finances this via borrowing, with Y and C unchanged. Recompute S, TB, and NCI. Interpret the direction of capital flows

Solutions

1.

$$
\begin{aligned}
S &= Y - C - G = 2000 - 1350 - 300 = 350, \\
TB &= x - im = 250 - 200 = 50, \\
I &= S - TB = 350 - 50 = 300, \\
NCI &= im - x = 200 - 250 = -50 \quad \text{(negative} \Rightarrow \text{net capital outflow)}.
\end{aligned}
$$

2.

$$
\begin{aligned}
I &= 330, \qquad S = 350 \\
TB &= S - I = 350 - 330 = 20, \\
NCI &= -TB = -20.
\end{aligned}
$$

3.

$$
\begin{aligned}
G &= 340, \qquad S = Y - C - G = 2000 - 1350 - 340 = 310, \\
TB &= S - I = 310 - 300 = 10, \\
NCI &= -10
\end{aligned}
$$

Practice Problem – Present Discounted Value

Assume a constant real interest rate $r > 0$ and deterministic cash flows $\{c_t\}_{t=1}^{T}$. Then:

$$PDV = \sum_{t=1}^{T} \frac{c_t}{(1+r)^t}$$

Suppose that $c_t = c_{t+1} \; \forall \; t$. Then:

$$PDV = \frac{c}{r}$$

1. A firm can buy a machine today for cost K. Starting next year it generates a constant real net cash flow c each year forever. The real interest rate is $r > 0$.
   (a) Compute the project's PDV and NPV (PDV - cost)
   (b) Derive the break-even condition in (c, r, K)
   (c) Numerical check: if $K = 900$, $r = 0.05$, and $c = 50$, should the firm invest in the machine?

(a) Because the cash flow and interest rate are constant, we can use the simplified PDV formula. As such, the NPV is:

$$NPV = \frac{c}{r} - K$$

(b) We want the minimum amount the project needs to return for the machine to be worth investing in. As such, we set the NPV to zero and solve for c:

$$\frac{c}{r} - K = 0$$
$$rK = c$$

(c) Plug-in the numbers:

$$NPV = \frac{50}{0.05} - 900$$
$$= 100$$

The firm should invest in the machine

Practice Problem – PDV and Opportunity Cost

You can either (i) invest K in a project that pays constant c forever starting next year, or (ii) place the same K in a risk-free asset that earns r per year forever.

1. Find the minimum $c^\star$ (as a function of K and r) that makes you indifferent between the project and the asset.
2. If r increases by $\Delta > 0$, what happens to $c^\star$? Explain the intuition in one sentence.
3. Numerical check: with $K = 1000$ and $r = 0.05$, compute $c^\star$. Recompute $c^\star$ if r rises to 0.06.

1. Option (1) gives us an NPV of $\frac{rK}{r} - K = 0$. Option (2) gives us an NPV of $\frac{c}{r} - K$. So set $NPV_1 = NPV_2$ and solve for c:

$$c^* = rK$$

2. c^* necessarily rises with r, since r multiplies K
3. Plug in the numbers:

$$c^* = 0.05(1000) = 50$$

In the second scenario:

$$c^* = 0.06(1000) = 60$$

Summary

- National savings equals investment plus the trade balance
- Net capital inflows equals -1 $\times$ the trade balance and denotes where assets flow
- Demand for investment is downward sloping in r
- Supply of savings is upward sloping in r
- The loanable funds market matches savings to investors

- Remember: homework due Friday night
- Read sections 10.2 - 10.3